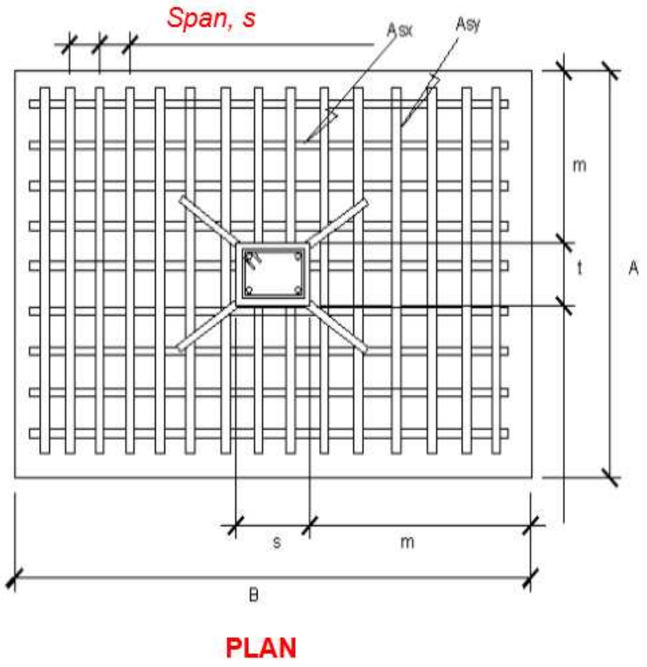
ISOLATED FOOTING DESIGN

CEng. William Rodríguez Serquén

1. The isolated footings, reinforced concrete structurals elements, that spread the column loads to the soil, so that, soil bearing capacity supports it. Soils of better bearing capacity, will use footings of minor sizes, respect to footings constructed in soils of less capacity.
2. The isolated footing design is the basis, for another kind of footings. Other footings have similar failure mechanism to isolated footings: Bending moment, shear, punching shear, development length and anchorage shall be verified. Also crushing strength must be verified.
3. Designing consists in calculate, shape and dimensions of concrete, as the steel quantity.
4. Data is needed previosly: axial load of the superestructure, cross section and reinforcement of the column that supports, and the soil bearing capacity (*q adm*), that supports the footing.



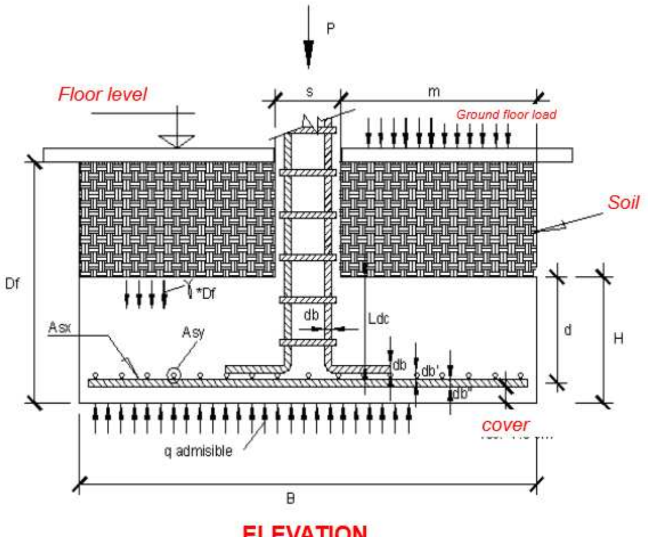


Fig.1. Elements for the isolate footing design. .

BASIC ELEMENTS AND UNITS TO USE

A, B = footing lengths, [m] or [ft]
s,t = column dimensions, [m] or [in]
m = footing cantilever length, [m] or [ft]
H = footing total depth (thickness), [m] or [ft]
P = axial load, [ton] or [kip]
q_{adm} *= admissible soil bearing capacity, [kg/cm²] or [lb/in²]*
L_d *= compressive development length (or in tension) of the column reinforcement, [cm] or [in]*
γ *= volumetric weight of the soil, [kg/m³] or [lb/ft³]*
Df = foundation depth, [m] or [ft]
gfl = ground floor load = 500 kg/m²
= 102.41lb/ft²
f´c = specified compressive strength of concrete, [kg/cm²] or [lb/in²]
fy =specified yield strength of reinforcement, [kg/cm²] or [lb/in²]
db, db´, db"= bar diameter of column, and footing reinforcement, [cm] or [ft]

5. We shall find the net stress (*q net*) that soil supports:

$$q\ net = q\ adm - \gamma * Df - gfl$$

Net stress means useful stress. The bearing capacity of a soil, is given to a foundation depth, D_f. Then, we must subtract the permanent loads that soil supports, above, such as filling load and ground floor load. The remaining soil resistance supports the superstructure.

6. We must calculate the total weight *Pt* of the superstructure that is transmitted to soil, including the weight of the isolated footing. Because of the

footing is not designed yet, the proportion obtained below can be useful.
We find the proportion *n*, between the footing weight *Pz* and the total (or service load) *P*, without factored loads, from the super structure, as function of net stress, concrete volumetric weight and the depth of the footing:

We define the proportion:

$$n = Pz / P,$$

We use the equilibrium law. The superstructure and footing load, equilibrate net stress of the soil multiplied by footing area:

$P + Pz = q\ net\ x\ A$, and

With the footing weight:

$Pz = \gamma_c * A * B * H,$

Where:
- γ_c = volumetric weight of reinforced concrete, *2400 kg/m³* (*149.8 lb/ft³*)
A, B, H = spans of the footing.
q net = net stress of the soil

We obtain: $$n = \frac{1}{\dfrac{q_{net}}{\gamma_c * H} - 1} \qquad \text{…(ZA-1)}$$

We find the footing weight P_f, as a fraction of total load *P*, trough:

P (footing weight) = n x P (service load)

The service load, is obtained as a total sum of loads from the superstructure. The coefficient named *n*, is obtained from the equation ZA-1 or from Fig. 2, where is tabulated.

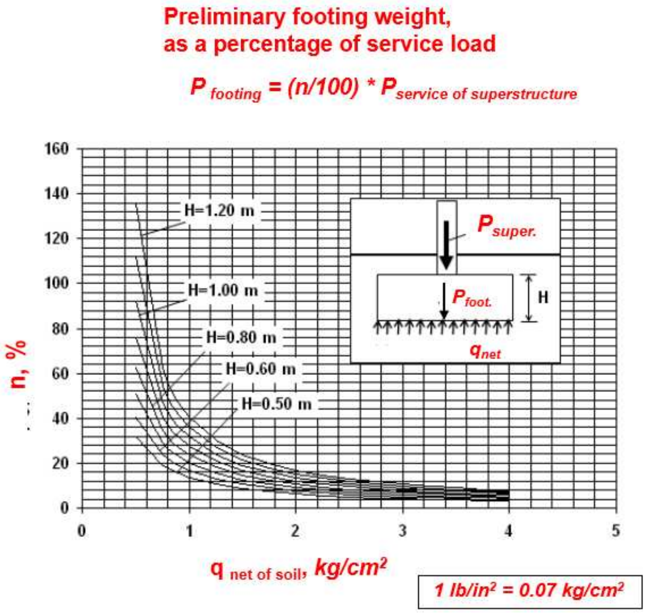


Fig. 2. Diagram for calculating preliminary isolated footing load, related with service load of the superstructure.

For the total load we use:

$Pt = P + (n)\ P,$

n is obtained from Fig. 2, or equation ZA-1.

7. We find the required footing area:

A footing = (*Pt*) /*q net*

8. As we look for similar cantiléver lenght:

$$(s + 2m)(t + 2m\ ) = A\ footing$$

Resolving the equation, we obtain *m,* or with the following approximate formula:

$$m = \ (\sqrt{Azap}\ / \ 2)\ \ - \ (s + t)/4$$

9. Then the footing dimensions *A* and *B* are:

$$A = 2m + t$$
$$B = 2m + s$$

$$\boldsymbol{A = \sqrt{(A\ footing)} - (s\text{-}t)/2}$$

$$\boldsymbol{B = \sqrt{(A\ footing)} + (s\text{-}t)/2}$$

Notice that if s = t, means, the column cross section is square, dimensions are equals, and is the squared root of the area.
As we notice, moments are not included. These moments exist generally in two directions, so they

increase the stresses on the soil in both directions, because of the excentricity that produces the addition of moments. The bidireccional flexure, is in the artícle: Mat slabs, of this author.

10. Then we must find the thickness of the footing *H*.
Footing thickness is calculated, after we find the effective depth "d", with the verification of:
-Development length
-Punching or bidirectional shear
-One-directional shear

11. The development length for compression is given for:

$ld = 0.08 * fy * db / \sqrt{f'c}$ …Norma ACI-318

$ld = 0.004\, db * fy$, or

$ld = 8\ in$, use the greatest value.

db = column steel diameter
db' = upper bar diameter of the footing slab grid
db'' = lower bar diameter of the footing slab grid

The "Building code requirements for structural concrete", book named ACI, specifies that, for seismic designs, the development length in tension also must be considered. In this paper, because this is a classical academic design, we will use the compressive development length.

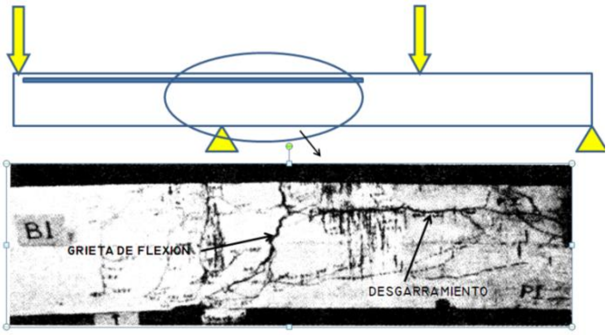


Fig.3. Failure for development lenght.

H is obtained with:

$H = ld + db` + db'' + covering.$

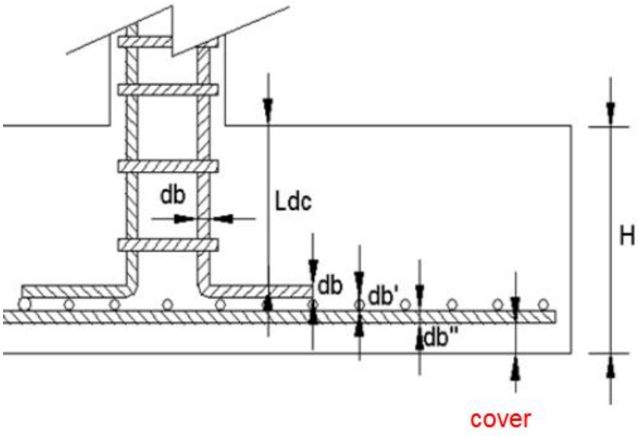


Fig. 4. Elements of total footing depth, *H*.

12. We must computate the ultimate soil reaction (qu):

$Pu = \alpha D + \beta L$ (Local code coefficients, of your country or state)

$Pu = 1.2\, D + 1.6\, L$ (ACI-318 Code)

D = dead load
L = live load

$qu = Pu / (A*B)$

13. The required punching stress is calculated with:

$Vp = Vu - 2* (s+d)\, (t+d)*d$

See the loads in Fig. 5, and its equivalent block in Fig. 6.

$-v\ required = Vp / (perímeter * d)$

$v\ required = qu* [\, A*B - (s+d)*(t + d)] / [2d*(s + t + 2*d)]$

….(A)

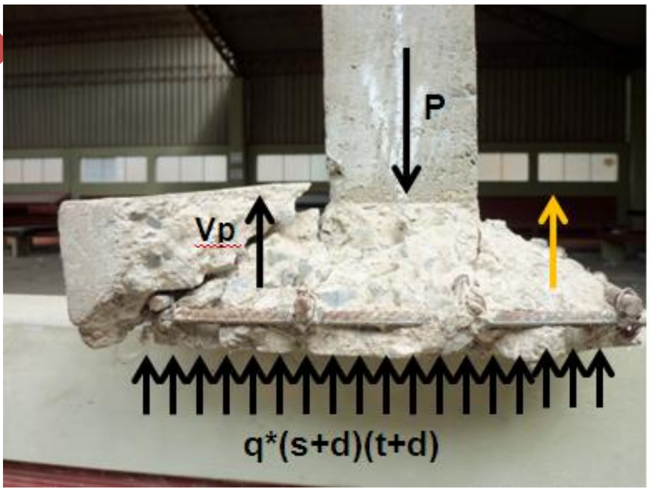
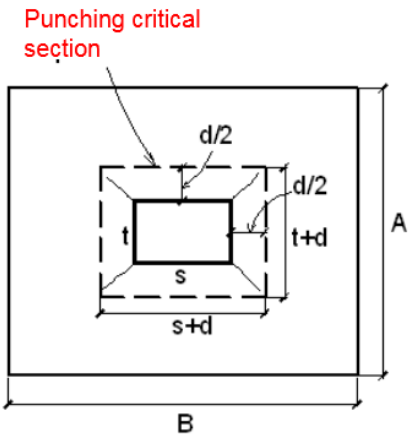
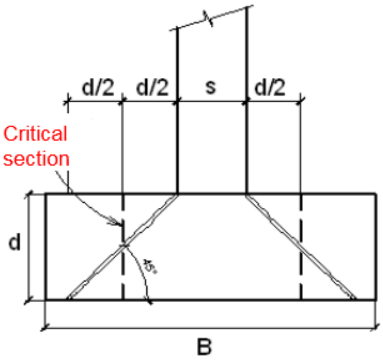


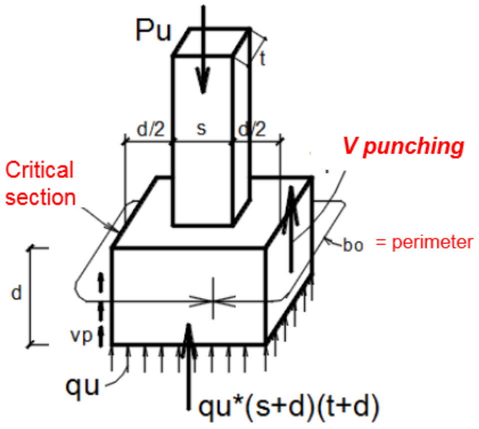
Fig. 5. Footing punching failure. Testing in Universidad Pedro Ruiz Gallo de Lambayeque. Perú.



PLAN



ELEVATION



EQUIVALENT BLOCK

Fig. 6. Block considered as equivalent to real punching failure.

14. The required punching strength must be less or equal to punching design strength:

$$v\ d = \phi * 0.27(2 + 4/\beta)\ \sqrt{f'c} \qquad \text{...Code ACI 318}$$

$\beta = s/t$ (major to minor size of the column)

Also:

$$v\ d = \phi * 1.1\ \sqrt{f'c}\ , \quad \phi = 0.85 \qquad \text{... ACI 318} \qquad \text{...(B)}$$

Wichever the less.

15. Depth "d_2" is obtained from:

$$(A) = (B)$$

$$q_u * [\ A*B - (s+d)*(t+d)] / [2d*(s+t+2*d)] = \begin{cases} \phi * 0.27(2 + 4/\beta)\ \sqrt{f'c} \\ \text{or} \\ \phi * 1.1\ \sqrt{f'c} \end{cases}$$

Be careful with units, must be the same. Generally is used:

$[A] = kilopound/foot^2$ *or tonelada* $/m^2$

$[B] = pound/inch^2$ *or kilogramo* $/cm^2$

16. The unidirectional shear is verified in the distance "d" from the column border. See Fig. 7 and Fig. 8.

Fig. 7. Sharing failure. Testing in the UNPRG. Lambayeque, Perú.

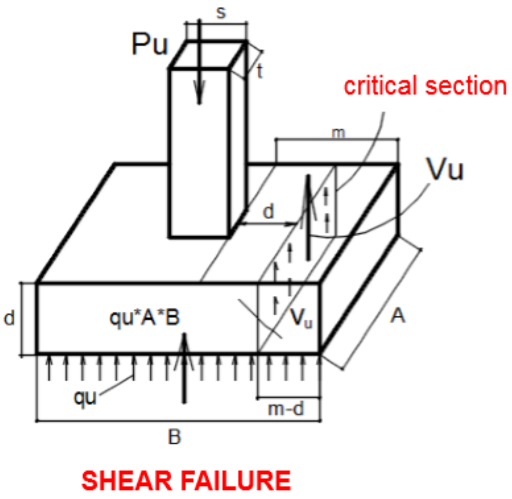


Fig. 8. Elements for calculating the sharing resistance.

In *x* direction:

*v required = qu *A*(m - d) /(A*d)*

In *y* direction:

*v required = qu *B*(m - d) / (B*d)*

They should be less or equal to shear design strength:

$$v\ adm = \phi * 0.53\ \sqrt{f'c}$$

$$\phi = 0.85$$

Then d_3 is obtained from:

$$qu*(m-d)/d = 0.85 * 0.53\ \sqrt{f'c}$$

17. From d_1, d_2 and d_3 we choose the maximum.

If $d_m = max\,(d1, d2, d3)$

$$H = d_m + db``/2 + cover$$

18. Crushing strength in column base shall be verified.-

Required stress fa, shall be less than design stress φ***fau*:**

$$f_a \leq \phi f_{au}$$

Required crushing stress, in the column basis:

$$f_a = \frac{P_u}{A_1}$$

Design crushing stress is:

$$f_{au} = \emptyset * \sqrt{\frac{A_2}{A_1}}(0.85\ f'_c) \qquad ....(a)$$

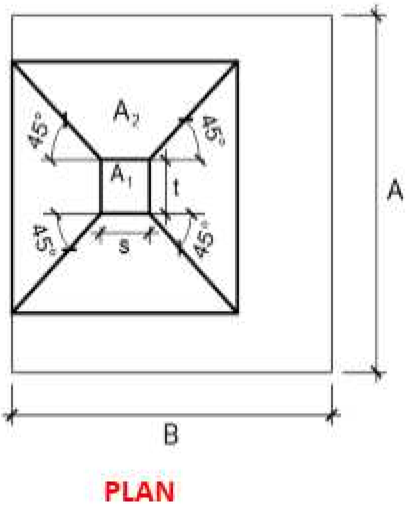
$$f_{au} = \emptyset * 2(0.85\ f'_c) \qquad ...(b)$$

$$\emptyset = 0.65$$

Whichever is minor from the two f_{au} values, obtained from (a) and (b).

.$f'c$ = footing concrete strength.

A_1, A_2 are obtained as showed below:



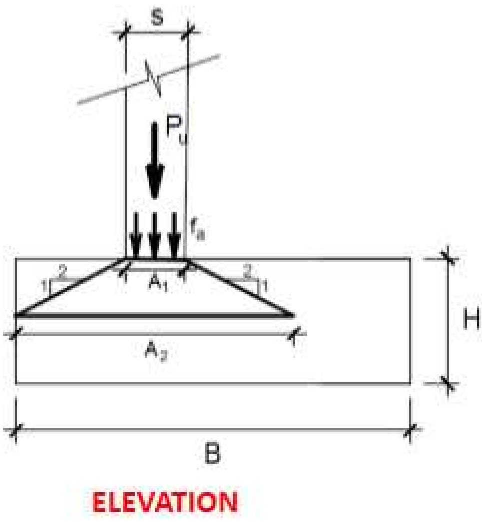


Fig. 9. Footing floor and elevation, shows the way to obtain A_1 and A_2. Both sections shall be concentric in floor.

If this equation is not satisfied:

$$f_a \leq \phi f_{au}$$

Steel bars shall be placed named dowels as showed:

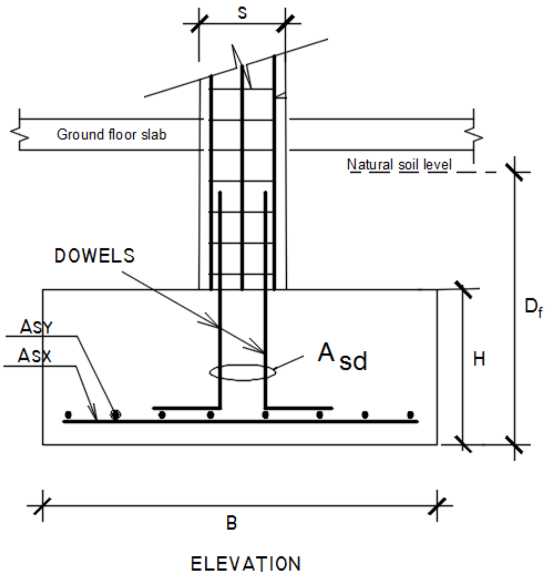


Fig. 10. Dowels placed to avoid bearing failure.

Dowells área is obtained this way:

$A_{sd} = \Delta F / (\varphi * fy)$

$\Delta F = (fa - fau)\, A_1$

.φ = 0.65 for compression dowells

.φ = 0.90 for tension dowells

Minimum steel area:

$Asd >= 0.005\, A_1$

19. Determination of **footing reinforcement** for bending moment**:**

Bending moment area is calculated, using the flexural moment produced by soil reaction in the column border.

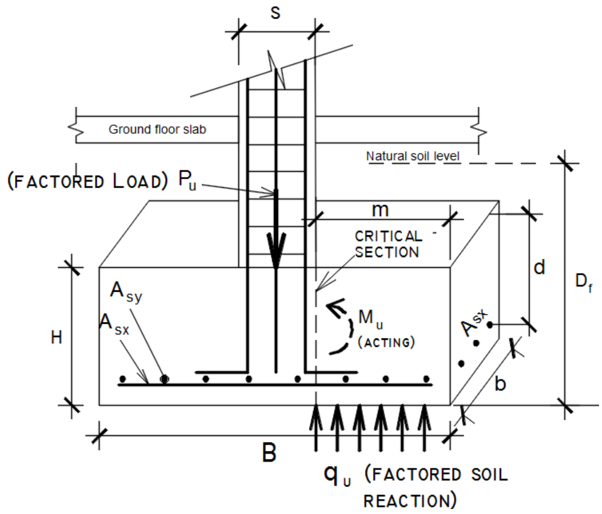


Fig. 11. Acting flexural moment in footing and elements for determining steel area.

For x direction:

$M_u = (q_u/2) * m^2 * B$

For y direction:

$M_u = (q_u/2) * m^2 * A$

We shall apply the following reinforced concrete equations for beams:

$\mathbf{A_s = M_u / [0.9 * f_y * (d - a/2)]}$

$\mathbf{a = As\, f_y / (0.85 * f'_c * b)}$

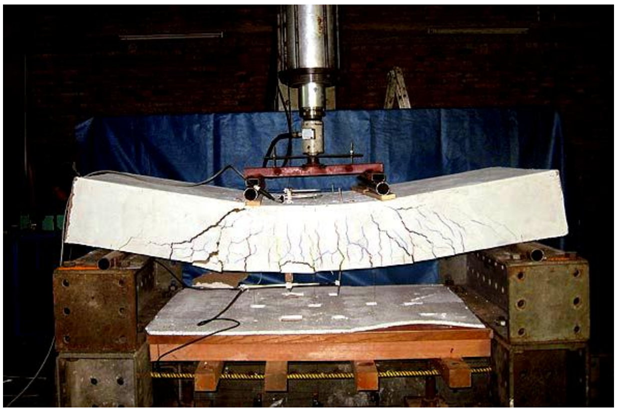


Fig. 12. Flexural failure of slab. Testing in Universidad Católica. Perú.

Fig. 13. Flexural failure of footing shows bi-directional deformation. Testing UNPRG. Perú.

20. Minimum reinforcement for flexural moment.-

Calculated steel area *As*, using formulas shall be greater than minimum reinforcement area *As min*:

$$-\rho\ min = 0.7\sqrt{fc}\ /\ fy$$

$$-As\ min = (0.7\sqrt{fc}\ /\ fy)\ b*d$$

Instead of minimum reinforcement in slabs is *0.0018,* minimum reinforcement for flexural members shall be used, regarding for footings that support stresses higher than temperature changes, such as punching and shearing stresses.

21. With reinforcement area, we shall find the bar number:

$Nb = As\ /\ Ab$

Ab = bar are

Number of bar, Nb	*Ab(cm^2)*
# 3 (3/8")	*0.71*
# 4 (1/2")	*1.29*
# 5 (5/8")	*2.00*
# 6 (3/4")	*2.84*
# 7(7/8")	*3.87*
# 8 (1")	*5.10*

With number of bar *Nb*, we determine the bar span *s* (see Fig. 14).

$(Nb-1)*s = B - 2*cover - db$

$$\mathbf{s = (B - 2*cover - db)\ /\ (Nb - 1)}$$

B = footing length
db = bar diameter
cover = *50 mm* or *75 mm*, it depends if concrete is or not in contact with soil.
Nb = used bar number

22. In our design, we shall use this notation:

Use 1 # bar @ s

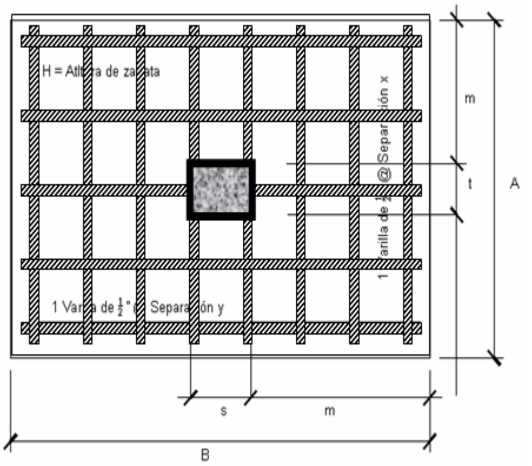
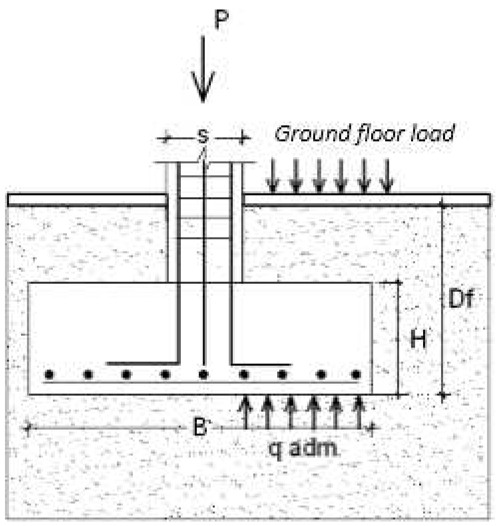
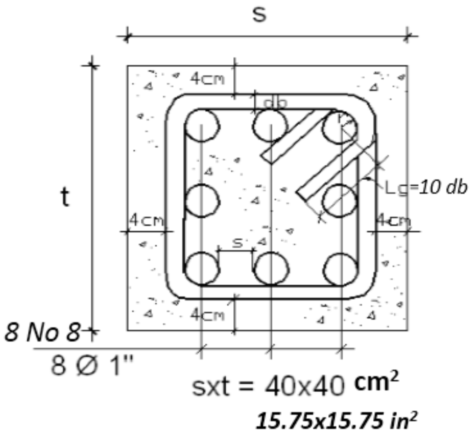


Fig. 14. Drawing of steel bars in isolated footing.

EXAMPLE OF ISOLATED FOOTING DESIGN

Let us design a reinforced concrete footing using the following parameters:





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Design parameters:

P_D= 140 t = 308.65 kip

P_L= 35 t = 77.16 kip

P = 140t + 35t= 175t = 385809 lb

$\gamma = 1.8\frac{t}{m^3} = 112.37\frac{lb}{ft^3}$ *, is volumetric weight of soil*

-q_{adm}= 1.5 kg/cm^2 = bearing capacity of soil

$$1.5\frac{kg}{cm^2} * \frac{1t}{1000kg} * \frac{1cm^2}{(10^{-2})^2 m^2} = 15\frac{t}{m^2} = 3072\frac{lb}{ft^2}$$

D_f =1.5m = 4.92ft, footing level depth

Ground floor load = 500 kg/m^2= 0.5 t/m^2
= 102.41 lb/ft^2

Column cross section:

sxt = 40 x 40 cm^2 =15.75x15.75 in^2
=1.31x1.31ft^2

$As = 8\_bars\_\#8$

$$f'c = 210\frac{kg}{cm^2} = 2.99\frac{kip}{ft^2}$$

$$fy = 4200\frac{kg}{cm^2} = 59.74\frac{kip}{ft^2}$$

$$e = 0.10m = 3.9in\_simple\_concrete\_depth$$

1. DETERMINATION OF FOOTING AREA.-

First, we find net stress of the soil:

$$q_{net} = q_{adm} - \gamma * D_f - sobrecarga\_de\_piso$$

$$q_{net} = 15\frac{t}{m^2} - (1.8\frac{t}{m^3} * 1.5m) - 0.5\frac{t}{m^2}$$

$$q_{net} = 3072\frac{lb}{ft^2} - (112.37\frac{lb}{ft^3} * 4.92ft) - 102.41\frac{lb}{ft^2} = 2417\frac{lb}{ft^2}$$

$$q_{net} = 11.8\frac{t}{m^2} = 1.18\frac{kg}{cm^2}$$

We determine the required footing area:

$$A_{foot} = \frac{(Pt)}{q_{net}}$$

$$A_{foot} = \frac{175t}{11.8\frac{t}{m^2}} = \frac{385809lb}{2417\frac{lb}{ft^2}}$$

$$A_{foot} = 14.83m^2 = 159.62\,foot^2$$

(s+2m)(t+2m) = $A_{footing}$

m is obtained solving the equation, approximately:

$$m = \left(\sqrt{A_{foot}}/2\right) - (s+t)/4$$

Then, footing lengths A and B are:

A = 2m+t
B = 2m+t

$$A = \left(\sqrt{A_{foot}}\right) - (s-t)/2$$

$$B = \left(\sqrt{A_{foot}}\right) + (s-t)/2$$

$$A = \left(\sqrt{A_{foot}}\right) = \sqrt{159.62\,ft^2} = 12.63\,foot$$
$$A = \left(\sqrt{A_{foot}}\right) = \sqrt{14.8\,m^2} = 3.85\,m$$

$$B = \left(\sqrt{A_{foot}}\right) = \sqrt{159.62\,ft^2} = 12.63\,foot$$
$$B = \left(\sqrt{A_{foot}}\right) = \sqrt{14.8\,m^2} = 3.85\,m$$

We accept footing lengths: 12,6x12,6 ft ^{2}

2. DETERMINATION OF FOOTING THICKNESS.-

Depth H is determined, calculating first, effective depth "d", verifying:

-Development length
-Punching
-Shear

2.1 Development length (cm).- Compression

$$Ld = 0.08 * \frac{f_y * db}{\sqrt{f'c}}$$

a. $$Ld = 0.08 * \frac{4200\,kg/cm^2 * 2.54\,cm}{\sqrt{210\,kg/cm^2}}$$
$$Ld = 58.59\,cm \cong 59\,cm = 23.07\_in$$

b. $$Ld = 0.004\,db * f_y$$
$$Ld = 0.004(2.54\,cm) * (4200\,kg/cm^2)$$
$$Ld = 42.7\,cm \cong 43\,cm = 16.81\_in$$

c. *Ld=20 cm =7.9 in*

Computate the factored soil reaction (q_u)

$P_u = 1.5\,P_D + 1.8\,P_L$

$P_u = 1.5(308647.2\ lb) + 1.8(77161.79\ lb)$

$P_u = 601862\ lb$

$P_u = 1.5(140\ t) + 1.8(35\ t)$

$P_u = 1.5(140\ t) + 1.8(35\ t)$

$P_u = 273\ t$

$q_u = P_u / A*B$

$$q_u = \frac{601862\ \text{lb}}{12.63\,ft * 12.63\,ft}$$
$$q_u = 3773\,\frac{lb}{ft^2}$$

$$q_u = \frac{273\,t}{3.85\,m * 3.85\,m}$$
$$q_u = 18.42\,\frac{t}{m^2}$$

2.2 Required bi-directional shear or punching stress is calculated:

$$-\ \text{v}_{\text{required}} = q_u * \frac{\left[A * B - (s+d)*(t+d)\right]}{2d * (s + t + 2*d)}$$

Required punching stress shall be less or equal to design strength:

$$-v_{design} = \phi * 0.27\left(2 + \frac{4}{\beta}\right) * \sqrt{f'c}\ ;$$

Where β, is the ratio of major to minor length of column

Also:

$$-v_{design} = \phi * 1.1\sqrt{f'c},\_\phi = 0.85$$

$$-\ v_{design} \begin{cases} \phi * 0.27\left(2 + \frac{4}{\beta}\right) * \sqrt{f'c} \\ \\ \phi * 1.1\sqrt{f'c} \end{cases}$$

$$\begin{cases} 0.85 * 0.27(2 + 4/1) * \sqrt{210} = 19.95\,\frac{kg}{cm^2} \\ \qquad = 283.8\,\frac{lb}{in^2} \end{cases}$$

$$0.85 * 1.1 * \sqrt{210} = 13.55\,\frac{kg}{cm^2}\ (\text{min})$$

$$= 192.7\,\frac{lb}{in^2}\ (\text{min})$$

$$q_u * [A * B - (s+d) * (t+d)] / (2d * (s+t+2*d)$$

$$= 192.7 \frac{lb}{in^2} = 27748.8 \frac{lb}{ft^2}$$

$$3773 \frac{lb}{ft^2} * [12.6 ft * 12.6 ft - (1.31 ft + d) * (1.31 ft + d)] /$$

$$2d * (1.31 ft + 1.31 ft + 2*d) = 27748.8 \frac{lb}{in^2}$$

$$d_2 = 1.71 ft$$

$$d_2 = 20.5 in$$

In *ton* and *m* units we have:

$$q_u * [A * B - (s+d) * (t+d)] / (2d * (s+t+2*d)$$

$$= 135.5 \frac{t}{m^2}$$

$$18.4 * [3.85 * 3.85 - (0.40 + d) * (0.40 + d)] /$$

$$2d * (0.4 + 0.4 + 2*d) = 135.5 \frac{t}{m^2}$$

$$d_2 = 0{,}52\_m$$

$$d_2 = 52\_cm$$

2.3 SHEAR DESIGN

REQUIRED SHEAR LOAD:

$$V_u = q_u (m-d) * A$$

REQUIRED SHEAR STRESS:

$$v_u = \frac{V_u}{A * d}$$

$$v_u = \frac{qu(m-d) * A}{A * d}$$

$$v_u = \frac{qu(m-d)}{d}$$

$$v_u = \frac{3791 \frac{lb}{ft^2} (5.659 ft - d)}{d}$$

DESIGN SHEAR STRESS:

$$v_d = 0.85 * 0.53 \sqrt{f'c}$$

$$v_d = 0.85 * 0.53 \sqrt{210}$$

$$v_d = 6.53 \text{ kg/cm}^2 = 13374.5 \text{ lb/ft}^2$$

$$13374.5 lb / ft^2 = \frac{3773 \frac{lb}{ft^2} (5.659 ft - d)}{d}$$

$$d_3 = 1.25 ft$$

$$d_3 = 14.9 in$$

In *ton* and *m* units we have:

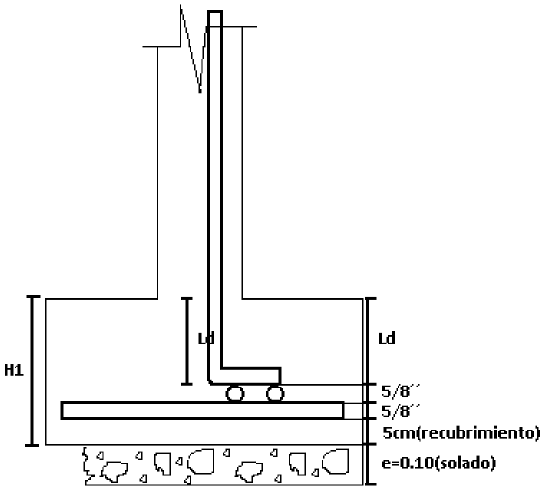
$$v_u = \frac{18.4 t / m^2 (1.725 - d)}{d}$$

$$65.3 t / m^2 = \frac{18.5 t / m^2 (1.725 - d)}{d}$$

$$d_3 = 0.38 m$$

$$d_3 = 38 cm$$

From depths d_1, d_2 and d_3, we choose the maximum value:



$$H = d + db' + db'' + recubrimiento$$

$$H = 23.07 in + 0.63 in + 0.63 in + 1.97 in$$

$$H = 26.3\_inch = 2.192 ft$$

Therefore *d* is:

$$d = 23.07 in + 0.63 in = 23.7 in = 1.975 ft$$

$$d = 60 \text{ cm}$$

Previously, bar No 5 is assumed as reinforcement at the bottom of footing.

$$H = d + db' + db'' + recubrimiento$$

$$H = 58.9 cm + \frac{5}{8} * 2.54 cm + \frac{5}{8} * 2.54 cm + 5 cm$$

$$H = 67.075 cm$$

VERIFICATION FOR CRUSHING STRENGTH

The required crushing stress is:

$$f_a = \frac{P_u}{A_1} = \frac{601862\ lb}{15.75x15.75\ in2} = 2426\ lb/in2$$

$$= 170.6\ kg/cm2$$

$$f_a = \frac{P_u}{A_1} = \frac{273\ 000\ kg}{40x40\ cm2} = 170.6\ kg/cm2$$

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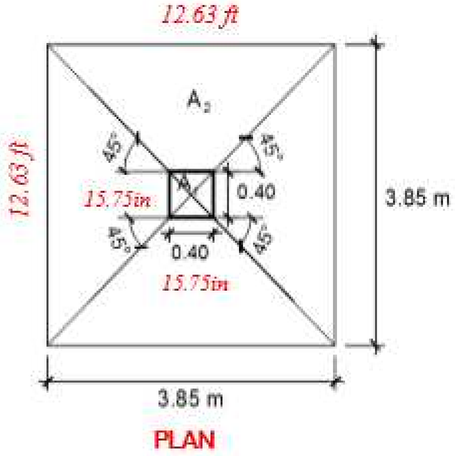


Design crushing stress is:

$$-f_{au} = \phi * \sqrt{\frac{A_2}{A_1}}(0.85\ f'_c) = \phi \sqrt{\frac{385x385}{40x40}}(0.85\ f'_c)$$

$$= \phi * 9.6\ (0.85\ f'_c) \qquad ...(a)$$

$$-f_{au} = \phi * 2(0.85\ f'_c) \qquad ...(b)$$

$\emptyset = 0.65$

We use the minimum value:

$$f_{au} = 0.65x2(0.85\ x210)\frac{kg}{cm2} = 232\frac{kg}{cm2}$$

The following equation is satisfied:

$$f_a \leq \phi f_{au}$$

Therefore, dowels are not needed.

3. DETERMINATION OF BENDING MOMENT REINFORCEMENT

Required moment strength, at the border of column:

$$M_u = \frac{q_u}{2} * m^2 * B$$

$$M_u = \frac{3773\dfrac{lb}{ft^2}}{2} * (5.659\,ft)^2 * 12.63\,ft$$

$$M_u = 763026.35 lb - ft$$

Lets use the last diagram, where we need use kg/cm2 units, to find the required reinforcement ratio, r:

$$\frac{M_u}{bd^2} = \frac{763026.35 lb - ft}{12.63\,ft * (1.975\,ft)^2}$$

$$\frac{M_u}{bd^2} = 15488.2\frac{lb}{ft^2} = 7.6\frac{lkg}{cm^2}$$

$$M_u = \frac{q_u}{2} * m^2 * B$$

$$M_u = \frac{18.42t / m^2}{2} * (1.725)^2 * 3.85$$

$$M_u = 105.5 * 10^5\,kg - cm$$

$$\frac{M_u}{bd^2} = \frac{105.51 * 10^5}{385 * 60^2}$$

$$\frac{M_u}{bd^2} = 7.6\frac{kg}{cm^2}$$

From the diagram given below, for f'c = 210 kg/cm2, we obtain:

$\rho = 0.002$, is the required reinforcement ratio for the footing. This must be greater than minimum ratio for flexural members:

$$\rho_{\min\_for\_flexural\_members} = 0.7\sqrt{f'c}/f_y ,nor$$

$$\rho_{\min} = 200/f_y\,(fy\_in\_psi)$$

$$\rho_{\min} = 14/f_y\,\_(fy\_in\_kg/cm^2)$$

$$\rho_{\min} = 14/4200 = 0{,}0033$$

$$Therefore\_we\_use:$$

$$\rho = 0.0033$$

$$A_s = \rho * b * d$$

$$A_s = 0.0033 * 151.6in * 23.7in$$

$$A_s = 11.86in^2$$

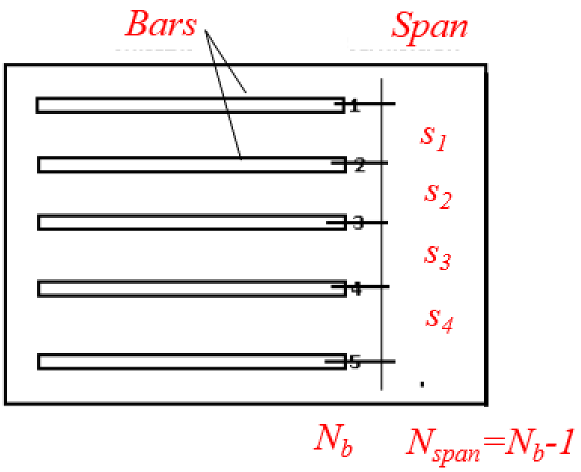
$$A_s = 76.52\_cm^2$$

We choose bar No 7 from the above table, and the area:

$$N_{bars} = 76.52 / 3.87$$

$$N_{bars} = 19.8$$

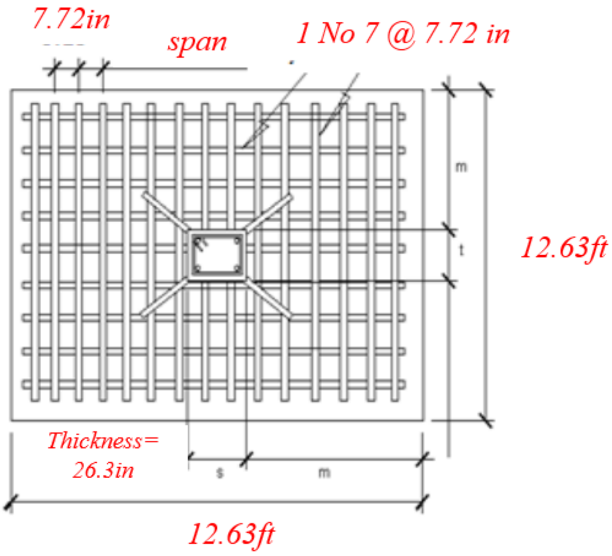
We use 20 bars



From the above figure, we deduce to obtain the bar span:

ABOUT THE FOOTING PLAN

Results are placed on a footing plan, with **Technical specifications**, that includes a summary of geotechnical investigations, concrete and reinforcement specifications, covers, concrete test, building code used for the design, provisions to: control salts in soil, avoid soil collapse, failure of neighbor buildings, because of our footing excavation, provisions about internal and external geodynamic phenomena than can affect the designed footing. Buildings and constructions generally cost a lot of money, and are supported by footings and designers must be aware of legal consequences, when footing collapses.



ISOLATED FOOTING AND REINFORCEMENT

DESIGN OF RECTANGULAR BEAMS WITH TENSION REINFORCEMENT